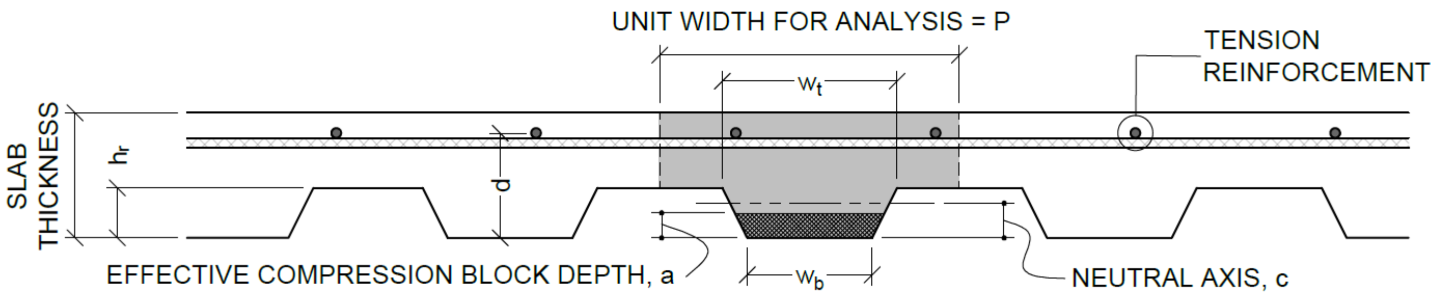
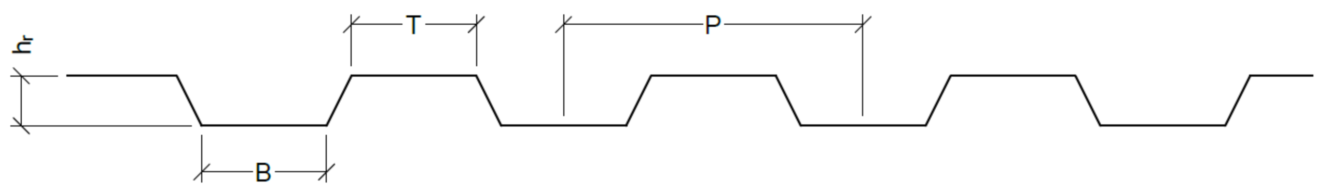
SLAB ON COMPOSITE DECKING: REINFORCEMENT FOR NEGATIVE BENDING OVER SUPPORTS EXAMPLE CALCULATIONS





Example Problem #1

INPUTS

- Two-span condition
- DL = 50 psf
- LL = 150 psf
- w_u = 1.2(50) + 1.6(150) = 300 psf
- Spacing of beams supporting slab = 10 feet
- 28-day compressive strength of concrete, f'_c = 5 ksi (β_1 = 0.80)
- Yield strength of WWR = 80 ksi
- Dimension to steel reinforcement centerline, d = 3.75 inches
- Slab thickness = 5 inches
- Deck rib height, h_r = 2 inches
- Deck dimension T = 5 inches
- Deck dimension B = 5 inches
- Deck dimension P = 12 inches
- Deck rib width at top w_t = 7 inches
- Deck rib width at bottom w_b = 5 inches
- Deck rib pitch ratio, horizontal over vertical = 0.5
- *Only tension-controlled solutions are considered valid.*

1. Calculate flexural demand.

$deck\ unit\ width\ for\ analysis = P = 12\ inches$

$$M_u = \frac{w_u l^2}{8} = \frac{300 \times 10^2}{8} = 3{,}750\ lb \cdot ft\ per\ foot = 3.75\ kip \cdot ft\ per\ foot$$

$$M_{n,required\ for\ analysis} = \frac{M_u}{\varphi} \times \frac{P}{12} = \frac{3.75}{0.9} \times \frac{12"}{12} = 4.167\ = 50\ kip \cdot in\ per\ analysis\ width$$

2. Determine maximum depth of effective compression block for tension-controlled behavior.

$$\varepsilon_y = \frac{f_y}{E} = \frac{80\ ksi}{29{,}000\ ksi} = 0.00276\, in/in$$

$$\varepsilon_s = \frac{0.003\beta_1 d}{a} - 0.003$$

$Set\ \varepsilon_s = 0.003 + \varepsilon_y\ (minimum\ strain\ corresponding\ to\ tension\ controllled\ behavior)$

$$0.003 +\ \varepsilon_y = \frac{0.003\beta_1 d}{a} - 0.003$$

$$a_{tension\ controlled} = \frac{0.003\beta_1 d}{\varepsilon_y + 0.006} = \frac{0.003 \times 0.80 \times 3.75"}{0.00276 + 0.006} = 1.027\ inches$$

$1.027 < h_r = 2$ ∴ $A\ tension\ controlled\ solution\ will\ have\ a\ trapezoidal\ compression\ block.$

3. Determine available moment strength if effective compression block depth is equal to deck rib height ($a = h_r$).

$$Centroid\ of\ trapezoidal\ shape, y' = \frac{h_r}{3} \times \frac{2w_t + w_b}{w_t + w_b} = 1.056", measured\ upward\ from\ w_b\ position$$

$$Area\ of\ trapezoidal\ shape = \frac{h_r \times (w_t + w_b)}{2} = 12\ in^2$$

$$M_{n,avail} = 0.85 \times f'_c \times A_{rib} \times arm$$

$$arm = d - y'$$

$$M_{n,avail} = 0.85 \times 5 \times 12 \times (3.75 - 1.056) = 137.4\ kip \cdot in$$

$where\ a = h_r: M_{n,avail} > \ M_{n,reqd\ for\ analysis}$

∴ $A\ solution\ with\ trapezoidal\ compression\ block\ exists.\ \ Must\ confirm\ that\ solution\ is\ tension\ controlled.$

4. Derive alternative equation forms for subsequent solving.

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$Where\ a\ trapezoidal\ area\ is\ defined\ by\ a\ depth\ equal\ to\ the\ effective\ compression\ block\ depth, a_{reqd}:$

$$A_{compression} = rib\ pitch \times a_{reqd}{}^2 + a_{reqd} \times w_b$$

$Defining\ the\ centroid\ distance\ of\ a\ trapezoidal\ shape\ with\ depth =\ a_{reqd}\ :$

$$y' = \frac{h_r}{3} \times \frac{2w_t + w_b}{w_t + w_b}\ where\ a = h_r$$

$$y' = \frac{a_{reqd}}{3} \times \frac{2w_t + w_b}{w_t + w_b}\ where\ a_{reqd} < h_r$$

The rib dimension w_b is a constant. For $a_{reqd} < h_r$, the "w_t" dimension is defined in terms of w_b.
The rib pitch is a constant.

$$w_t = \left[w_b + \left(2 \times a_{reqd} \times rib\ pitch\right)\right]$$

$$y' = \frac{a_{reqd}}{3} \times \frac{2\left[w_b + \left(2 \times a_{reqd} \times rib\ pitch\right)\right] + w_b}{\left[w_b + \left(2 \times a_{reqd} \times rib\ pitch\right)\right] + w_b}$$

$$y' = \frac{a_{reqd}}{3} \times \frac{2w_b + 4\left(a_{reqd} \times rib\ pitch\right) + w_b}{2w_b + 2\left(a_{reqd} \times rib\ pitch\right)}$$

$$y' = \frac{3\left(a_{reqd}\right)\left(w_b\right) + 4\left(a_{reqd}{}^2\right)\left(rib\ pitch\right)}{6\left[w_b + \left(a_{reqd}\right)\left(rib\ pitch\right)\right]}$$

$With\ constants\ w_b = 5\ inches\ and\ rib\ pitch = 0.5:$

$$y' = \frac{3\left(a_{reqd}\right)(5) + 4\left(a_{reqd}{}^2\right)(0.5)}{6\left[5 + \left(a_{reqd}\right)(0.5)\right]} = \frac{15a_{reqd} + 2a_{reqd}{}^2}{30 + 3a_{reqd}}$$

5. Determine a_{reqd}.

$$M_{n,reqd} = 50\ kip \cdot in = 0.85 \times f'_c \times A_{compression} \times (d - y')$$

$$50\ kip \cdot in = 0.85 \times 5\ ksi \times \left(0.5 \times a_{reqd}{}^2 + a_{reqd} \times w_b\right) \times \left[d - \frac{15a_{reqd} + 2a_{reqd}{}^2}{30 + 3a_{reqd}}\right]$$

$$50\ kip \cdot in = 0.85 \times 5\ ksi \times \left(0.5 \times a_{reqd}{}^2 + a_{reqd} \times 5"\right) \times \left[3.75 - \frac{15a_{reqd} + 2a_{reqd}{}^2}{30 + 3a_{reqd}}\right]$$

$By\ iteration\ or\ quadratic, solve\ for\ a_{reqd} = 0.646\ inches.$

$a_{reqd} = 0.646\ inches < a_{tension\ controlled} = 1.027\ inches\ \therefore$ ***solution is tension controlled!***

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6. Determine minimum required steel reinforcement area per analysis width.

$$A_s f_y = 0.85 f'_c A_{compression}$$

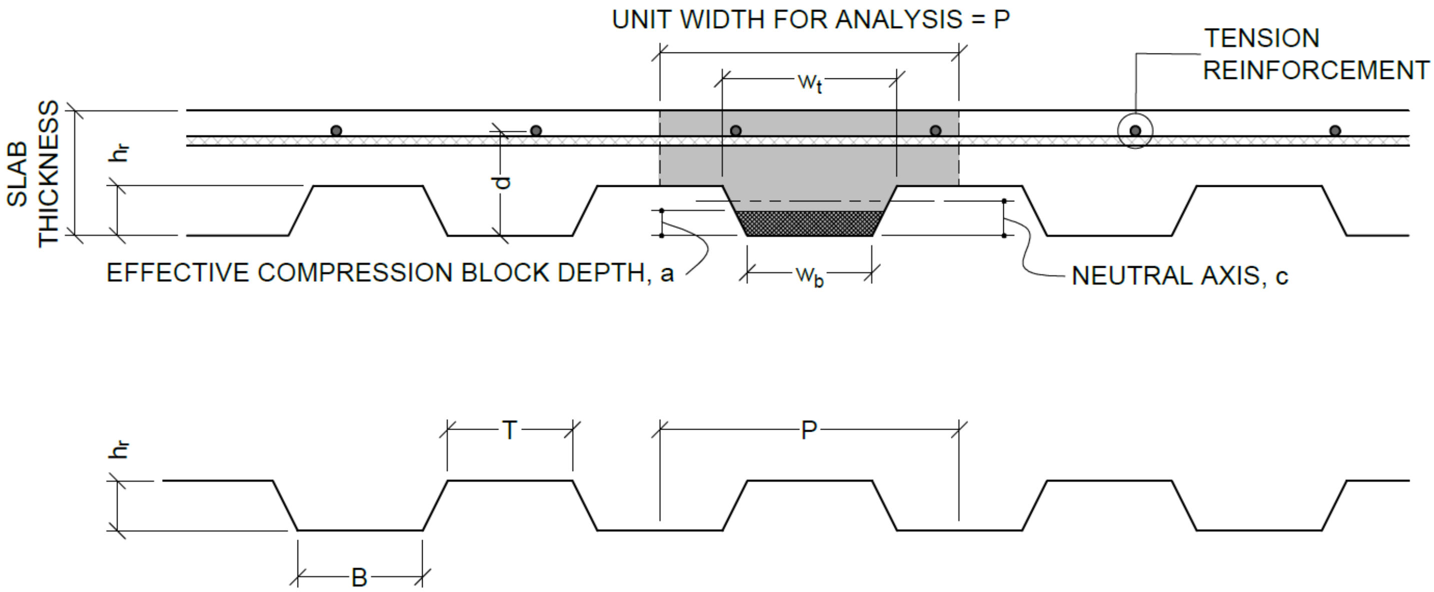
$$A_s = \frac{0.85 f'_c A_{compresion}}{f_y} = \frac{0.85 \times 5 \times 3.439\ in^2}{80} = 0.183\ in^2\ per\ analysis\ width$$

7. Determine minimum required steel reinforcement area per one foot unit width.

$$0.183\ in^2\ per\ analysis\ width \times \frac{12}{12"\ analysis\ width} = 0.183\ in^2\ per\ foot\ unit\ width$$

8. Derive WWR options based on user selected wire spacing.

User defined spacing (inches)	Minimum required wire size	Steel area achieved
6	D9.2	0.183 in^2 per foot unit width
8	D12.2	
10	D15.3	
12	D18.3	



Example Problem #2

INPUTS

- Two-span condition
- DL = 150 psf
- LL = 400 psf
- w_u = 1.2(150) + 1.6(400) = 820 psf
- Spacing of beams supporting slab = 10 feet
- 28-day compressive strength of concrete, f'_c = 3 ksi (β_1 = 0.85)
- Yield strength of WWR = 80 ksi
- Dimension to steel reinforcement centerline, d = 8 inches
- Slab thickness = 9 inches
- Deck rib height, h_r = 1.5 inches
- Deck dimension T = 3.5 inches
- Deck dimension B = 1.75 inches
- Deck dimension P = 6 inches
- Deck rib width at top w_t = 2.5 inches
- Deck rib width at bottom w_b = 1.75 inches
- Deck rib pitch ratio, horizontal over vertical = 0.25
- *Only tension-controlled solutions are considered valid.*

1. Calculate flexural demand.

$$deck\ unit\ width\ for\ analysis = P = 6\ inches$$

$$M_u = \frac{w_u l^2}{8} = \frac{820 \times 10^2}{8} = 10{,}250\ lb \cdot ft\ per\ foot = 10.25\ kip \cdot ft\ per\ foot$$

$$M_{n,required\ for\ analysis} = \frac{M_u}{\varphi} \times \frac{P}{12} = \frac{10.25}{0.9} \times \frac{6"}{12} = 5.694\ = 68.33\ kip \cdot in\ per\ analysis\ width$$

2. Determine maximum depth of effective compression block for tension-controlled behavior.

$$\varepsilon_y = \frac{f_y}{E} = \frac{80\ ksi}{29{,}000\ ksi} = 0.00276\ in/in$$

$$\varepsilon_s = \frac{0.003\beta_1 d}{a} - 0.003$$

$$Set\ \varepsilon_s = 0.003 + \varepsilon_y\ (minimum\ strain\ corresponding\ to\ tension\ controllled\ behavior)$$

$$0.003 +\ \varepsilon_y = \frac{0.003\beta_1 d}{a} - 0.003$$

$$a_{tension\ controlled} = \frac{0.003\beta_1 d}{\varepsilon_y + 0.006} = \frac{0.003 \times 0.85 \times 8"}{0.00276 + 0.006} = 2.329\ inches$$

$$2.329"\ > \ \mathrm{h_r} = 1.5"\ \therefore A\ tension\ controlled\ solution\ could\ have\ a\ trapezoidal\ tee\ compression\ block.$$

3. Determine available moment strength if effective compression block depth is equal to deck rib height (a = h_r).

$$Centroid\ of\ trapezoidal\ shape, y' = \frac{h_r}{3} \times \frac{2w_t + w_b}{w_t + w_b} = 0.794", measured\ upward\ from\ w_b\ position$$

$$Area\ of\ trapezoidal\ shape = \frac{h_r \times (w_t + w_b)}{2} = 3.1875\ in^2$$

$$M_{n,avail} = 0.85 \times f'_c \times A_{rib} \times arm$$

$$arm = d - y'$$

$$M_{n,avail} = 0.85 \times 3 \times 3.1875 \times (8 - 0.794) = 58.57\ kip \cdot in\ per\ analysis\ width$$

$$where\ a = h_r\!: M_{n,avail} = 58.57 <\ M_{n,reqd\ for\ analysis} = 68.33$$

$$\therefore A\ solution\ with\ trapezoidal\ tee\ compression\ block\ exists.\ \ Must\ confirm\ that\ solution\ is\ tension\ controlled.$$

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4. Determine a_{reqd}.

$M_{n,reqd} = 68.33\ kip \cdot in = M_{n,a=h_r} + M_{n,rectangular}$

$Where\ M_{n,rectangular}\ is\ defined\ by\ a\ rectangular\ section\ of\ slab\ above\ the\ rib\ of\ unit\ width\ and\ thickness.$

$$M_{n,reqd} = 68.33\ kip \cdot in = M_{n,a=h_r} + 0.85 \times f'_c \times unit\ width\ \times thickness \times \left(d - h_r - \frac{thickness}{2}\right)$$

$$M_{n,reqd} = 68.33\ kip \cdot in = 58.57 + 0.85 \times f'_c \times unit\ width\ \times thickness \times \left(d - h_r - \frac{thickness}{2}\right)$$

$$M_{n,reqd,rectangular\ section\ contribution} = 68.33 - 58.57 = 9.76\ kip \cdot in$$

$$9.76\ kip \cdot in = 0.85 \times f'_c \times unit\ width\ \times thickness \times \left(d - h_r - \frac{thickness}{2}\right)$$

$$9.76\ kip \cdot in = 0.85 \times 3\ ksi \times 6"\ \times thickness \times \left(8" - 1.5" - \frac{unit\ thickness}{2}\right)$$

$Solve\ for\ the\ thickness\ of\ the\ rectangular\ component\ of\ the\ compression\ section = 0.09893\ inches.$

$a_{reqd} = h_r + rectangular\ thickness = 1.5" + 0.09893" = 1.599\ inches$

$a_{reqd} = 1.599\ inches < a_{tension\ controlled} = 2.329\ inches\ \therefore$ ***solution is tension controlled!***

5. Determine minimum required steel reinforcement area per analysis width.

$T = A_s f_y = C_{rib} + C_{rectangle}$

$A_s f_y = 0.85 f'_c A_{rib} + 0.85 f'_c A_{rectangle}$

$A_s \times 80 = (0.85 \times 3 \times 3.1875) + [0.85 \times 3 \times (0.09893 \times 6)]$

$A_s \times 80 = 0.85 \times 3 \times 3.1875 + 0.85 \times 3 \times (0.09893 \times 6)$

$A_s = 0.121\ in^2\ per\ analysis\ width$

6. Determine minimum required steel reinforcement area per one foot unit width.

$$0.121\ in^2\ per\ analysis\ width \times \frac{12}{6"\ analysis\ width} = 0.242\ in^2\ per\ foot\ unit\ width$$

7. Derive WWR options based on user selected wire spacing.

User defined spacing (inches)	Minimum required wire size	Steel area achieved
6	D12.1	0.242 in^2 per foot unit width
8	D16.2	
10	D20.2	
12	D24.2	