January 2022

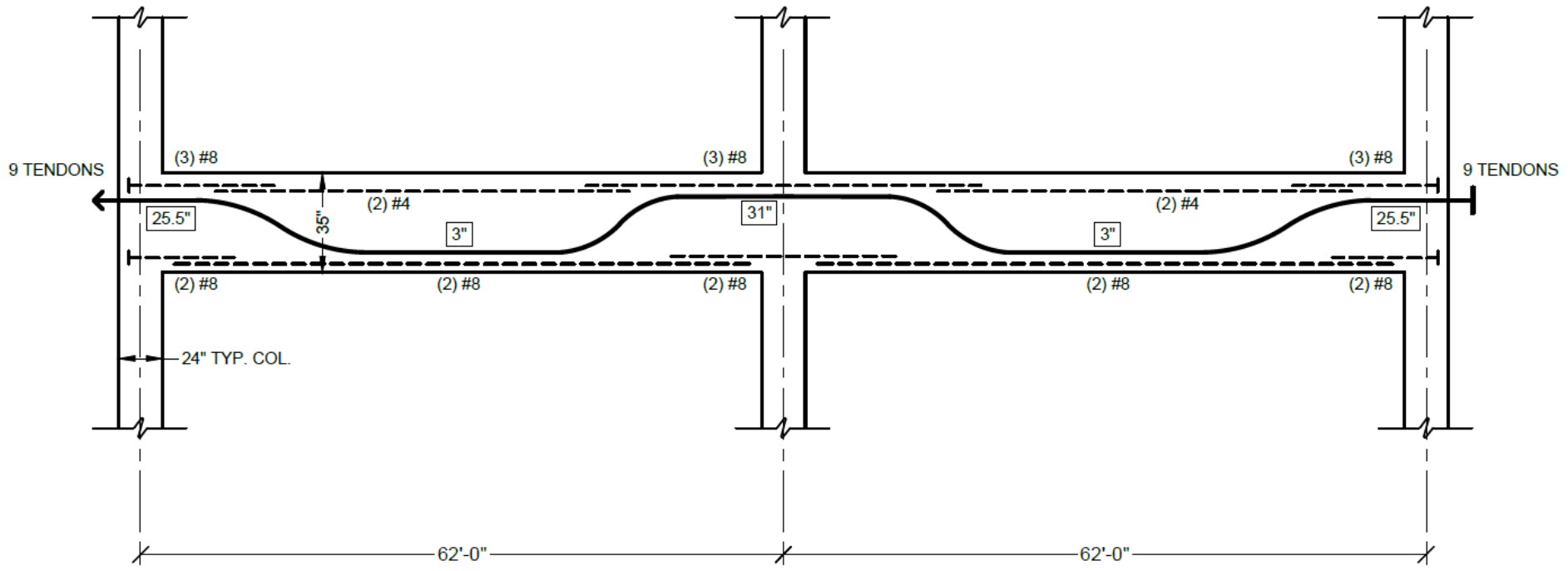
Technical Blog

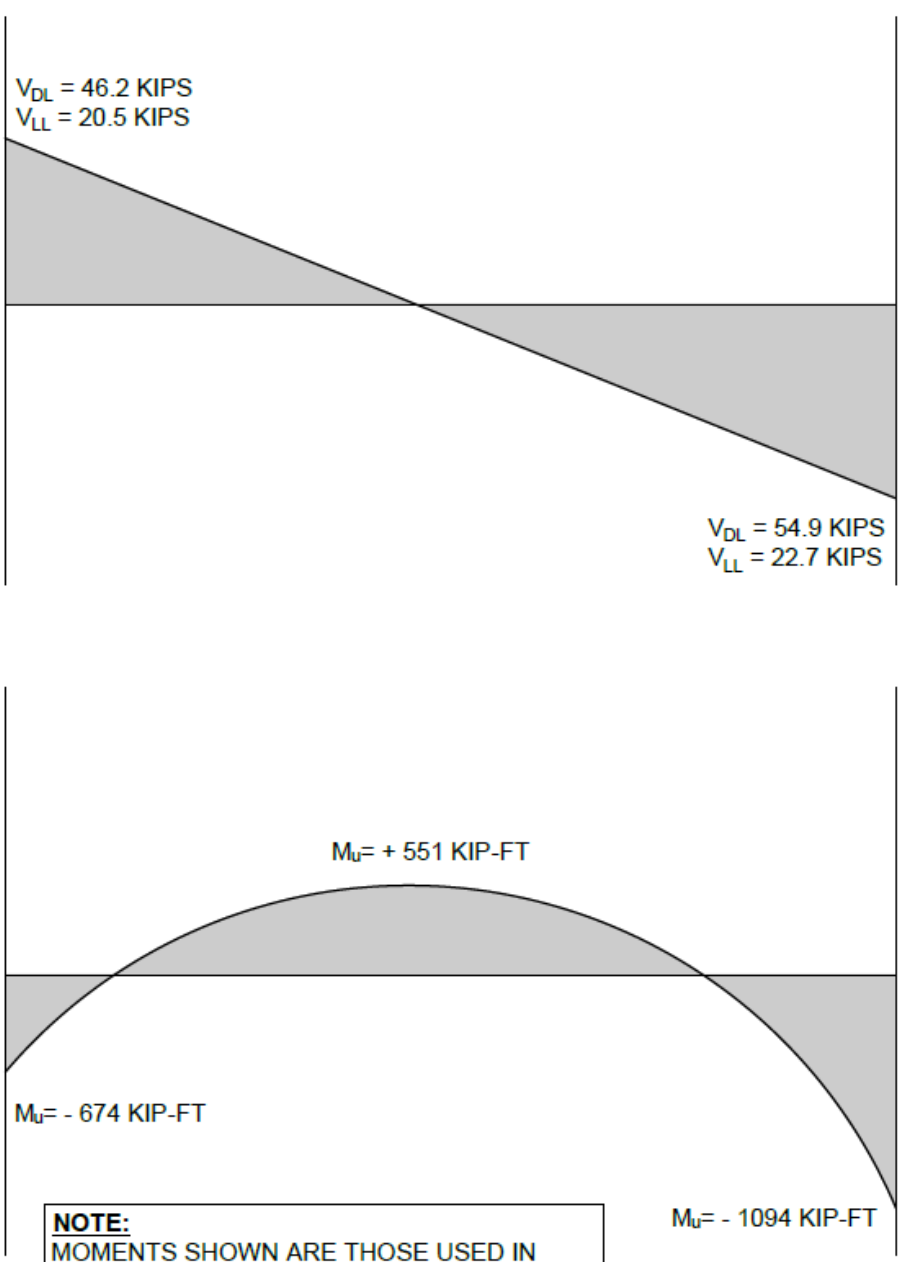
Design Spotlight: WWR Shear Reinforcement in a Parking Garage Post-Tensioned Beam, Part I

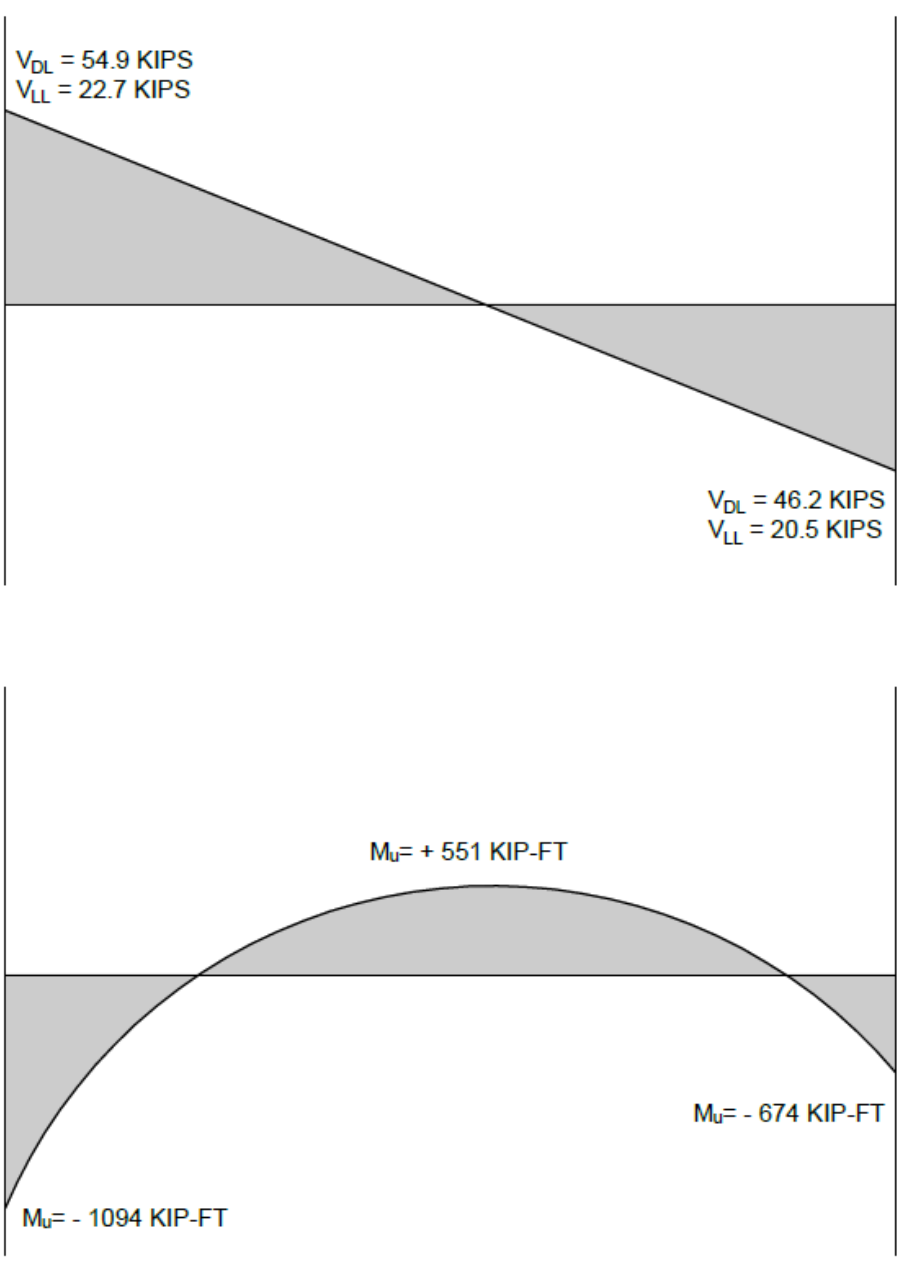
In this example, the structural engineer is tasked with the design of shear reinforcement for a post-tensioned beam in a parking garage structure. The design inputs are as follows:

- Post-tensioned concrete beam with two spans of 62'-0" dimension between centerlines of three (3) 24" x 24" columns.
- Beam is 30 inches deep with 14-inch width at its bottom surface and 16-inch width at the underside of a 5-inch thick slab. Beam spacing is 18 feet on center.
- Superimposed dead load is 5 psf. Superimposed uniform live load is 40 psf.
- 28-day concrete compressive strength is 4,500 psi.
- Shear reinforcement yield strength f_{yt} = 80,000 psi, assuming use of WWR.
- Reinforcing bars used as bonded longitudinal beam reinforcement have a yield strength equal to 60 ksi. Clear cover dimensions to rebar, used in flexural design = 3" bottom and 3" top.
- Post-tensioning tendons have the following properties:
    - f_{pu} = 270 ksi (specified tensile strength of prestressing reinforcement)
    - f_{se} = 174 ksi (effective stress in prestressed reinforcement, after allowance for all prestress losses)
    - A_{ps} = 0.153 in^2 *per tendon*

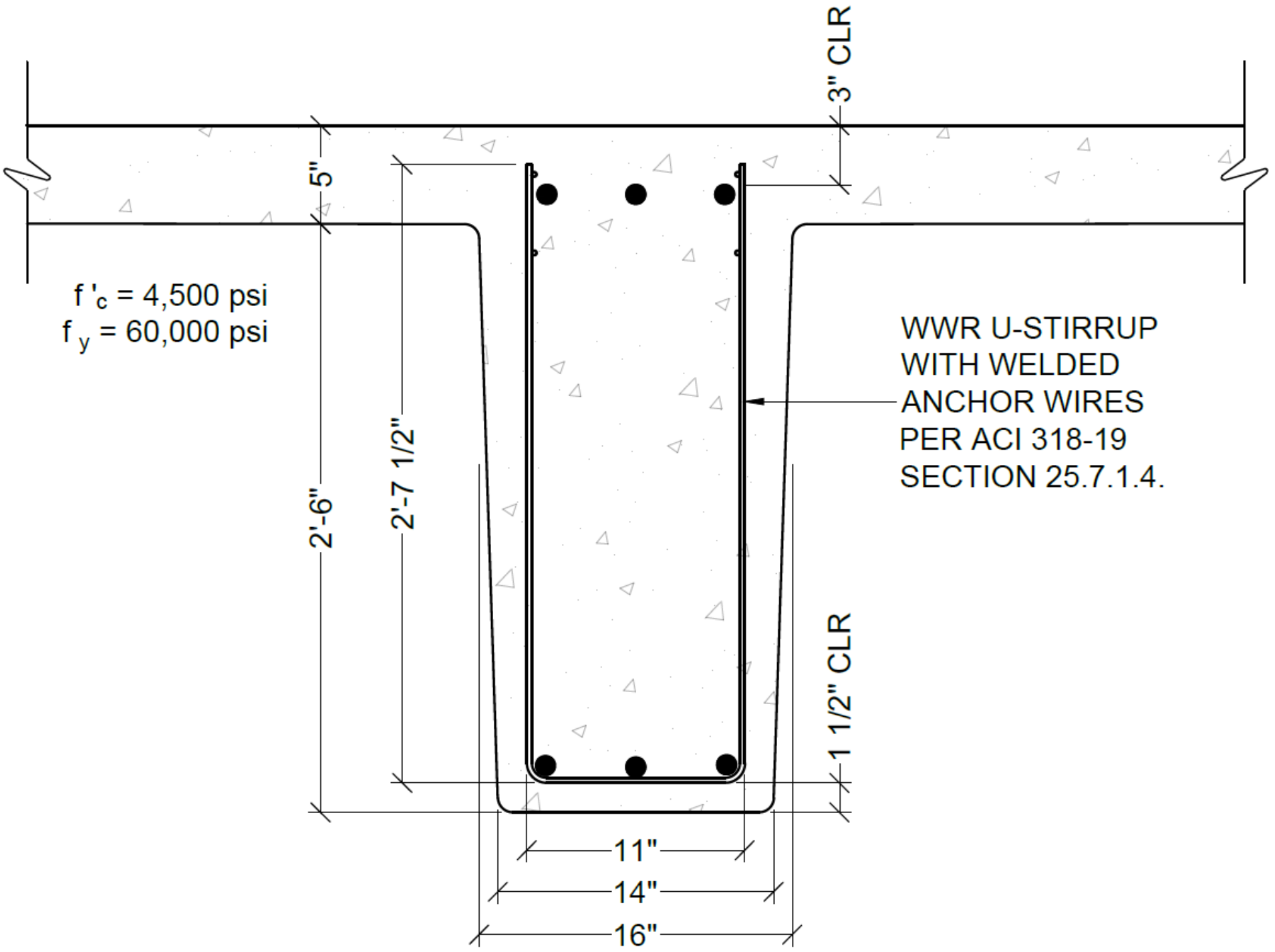
The resulting beam flexural design is shown below.







NOTE:
MOMENTS SHOWN ARE THOSE USED IN SHEAR DESIGN, AND IGNORE SECONDARY MOMENTS DUE TO THE EFFECTS OF THE POST-TENSIONING. *(FLEXURAL DESIGN, NOT SHOWN HERE, REQUIRES INCLUSION OF SECONDARY MOMENTS.)*



Step 1 – Confirm WWR material suitability

ACI 318-19 Table 20.2.2.4(a) confirms that ASTM A1064 WWR with f_{yt} = 80 ksi suitable for stirrups, ties, hoops in all shear design applications.

Step 2 – Establish requirements related to design strength calculation

ACI 318-19 Section 9.5.1.1:	Need $\varphi V_n \geq V_u$
Section 9.5.3.1:	V_n shall be calculated in accordance with Section 22.5
Section 22.5.1.1:	$V_n = V_c + V_s$
Section 22.5.1.4:	For prestressed members, calculate design strength V_c per Section 22.5.6 or 22.5.7.
Section 22.5.1.6:	V_s shall be calculated in accordance with Section 22.5.8.
Section 22.5.1.2:	Need $V_u \leq \varphi\left(V_c + 8\sqrt{f'_c}\, b_w d\right)$
Section 22.5.2.1:	For calculation of V_c and V_s, "d" need not be less than 0.8h.
Section 21.2.1:	For shear action, $\varphi = 0.75$.

Step 3 – determine applicability of ACI 318-19 Table 22.5.6.2.

ACI 318-19 Table 22.5.6.2 provides for an approximate method of calculating V_c but needs to be confirmed as applicable prior to proceeding.

If $A_{ps}f_{se} \geq 0.4\left(A_{ps}f_{pu} + A_s f_y\right)$, then V_c can be calculated in accordance with Table 22.5.6.2.

$A_{ps} = 9 \times 0.153\ \text{in}^2 = 1.377\ \text{in}^2$

$f_{se} = 174\ \text{ksi}$

$f_{pu} = 270\ \text{ksi}$

$A_s = 3 \times 0.79\ \text{in}^2 = 2.37\ \text{in}^2$

$f_y = 60\ \text{ksi}$

$\rightarrow 1.377 \times 174 = 240\ \text{kips} \geq 0.4(1.377 \times 270 + 2.37 \times 60) = 206\ \text{kips}$ $\therefore$ calculate V_c in accordance with Table 22.5.6.2.

Step 4 – Calculate shear demand and concrete shear capacity

Shear capacity shall be per Table 22.5.6.2, as below.

V_c shall be the least of the following, but need not be less than $2\lambda\sqrt{f'_c}b_w d$:

(a) $\left(0.6\lambda\sqrt{f'_c} + 700\frac{V_u d_p}{M_u}\right) b_w d$

(b) $\left(0.6\lambda\sqrt{f'_c} + 700\right) b_w d$

(c) $5\lambda\sqrt{f'_c} b_w d$

A summary of results for the left span is shown below. The right span is identical but mirrored.

From centerline (ft)	V_u (kips)	d = 0.8h (in)	Mu (kip-ft)	d_p (in)	b_w (in)	V_c (a)	V_c (b)	V_c (c)	V_c (min)	V_c (kips)
1	**88.2**	28	-673.8	11.01	14	-17.2	290.2	131.5	52.6	**52.6**
2.46 (h/2 from face)	**83.6**	28	-548.6	13.13	14	-30	290.2	131.5	52.6	**52.6**
3	**81.9**	28	-503.8	13.88	14	-35.9	290.2	131.5	52.6	**52.6**
5	**75.5**	28	-346.5	16.54	14	-66.7	290.2	131.5	52.6	**66.7**
7	**69.2**	28	-201.9	18.99	14	-133.1	290.2	131.5	52.6	**131.5**
9	**62.8**	28	-70	21.23	14	-419.8	290.2	131.5	52.6	**131.5**
11	**56.5**	28	49.3	23.26	14	625.4	290.2	131.5	52.6	**131.5**
13	**50.1**	28	155.9	25.08	14	200.1	290.2	131.5	52.6	**131.5**
15	**43.8**	28	249.7	26.69	14	122.9	290.2	131.5	52.6	**122.9**
17	**37.5**	28	330.9	28.09	14	88.6	290.2	131.5	52.6	**88.6**
19	**31.1**	28	399.4	29.28	14	68	290.2	131.5	52.6	**68**
21	**24.8**	28	455.2	30.26	14	53.5	290.2	131.5	52.6	**53.5**
23	**18.4**	28	498.2	31.03	14	42	290.2	131.5	52.6	**52.6**
25	**12.1**	28	528.7	31.59	14	32.4	290.2	131.5	52.6	**52.6**
27	**5.7**	28	546.4	31.93	14	23.4	290.2	131.5	52.6	**52.6**
29	**0.7**	28	551.4	32	14	16.8	290.2	131.5	52.6	**52.6**
31	**7.1**	28	543.7	32	14	25.4	290.2	131.5	52.6	**52.6**
33	**13.4**	28	523.4	31.72	14	34.4	290.2	131.5	52.6	**52.6**
35	**19.7**	28	490.3	31.22	14	44.5	290.2	131.5	52.6	**52.6**
37	**26.1**	28	444.6	30.52	14	56.8	290.2	131.5	52.6	**56.8**
39	**32.4**	28	386.2	29.61	14	72.6	290.2	131.5	52.6	**72.6**
41	**38.8**	28	315	28.49	14	96.1	290.2	131.5	52.6	**96.1**
43	**45.1**	28	231.2	27.15	14	136.9	290.2	131.5	52.6	**131.5**
45	**51.5**	28	134.7	25.61	14	239.7	290.2	131.5	52.6	**131.5**
47	**57.8**	28	25.6	23.85	14	1247.2	290.2	131.5	52.6	**131.5**
49	**64.2**	28	-96.4	21.89	14	-317.6	290.2	131.5	52.6	**131.5**
51	**70.5**	28	-231	19.72	14	-121.9	290.2	131.5	52.6	**121.9**
53	**76.8**	28	-378.2	17.33	14	-64.7	290.2	131.5	52.6	**64.7**
55	**83.2**	28	-538.2	14.74	14	-36.4	290.2	131.5	52.6	**52.6**
57	**89.5**	28	-710.8	11.93	14	-18.6	290.2	131.5	52.6	**52.6**
59	**95.9**	28	-896.2	8.92	14	-6.1	290.2	131.5	52.6	**52.6**
59.54	**97.6**	28	-948.5	8.06	14	-3.2	290.2	131.5	52.6	**52.6**
61	**102.2**	28	-1094.2	5.69	14	3.7	290.2	131.5	52.6	**52.6**

(Note that ACI 318-19 Section 22.5.1.2 is satisfied at all locations.)

Step 5 – Determine shear reinforcement requirements

From ACI 318-19 Section 9.6.3.2 for prestressed beams, and because the beam in this example does not qualify for one of the conditions listed in Table 9.6.3.1:

Provide $A_{v,min}$ in all regions where $V_u \geq 0.5\varphi V_c$.

And for this beam, per relationship noted in Step 3 and using 80 ksi WWR, ACI 318-19 Table 9.6.3.4 requires:

$\frac{A_{v,min}}{s}$ is the lesser of	the greater of:	$0.75\sqrt{f'_c}\frac{b_w}{f_{yt}} = 0.0088$
		$50\frac{b_w}{f_{yt}} = 0.00875$
	$\frac{A_{ps}f_{pu}}{80f_{yt}d} \times \sqrt{\frac{d}{b_w}} = 0.00293$	

With this, we will have the following shear regions to identify:

1. Where $V_u < 0.5\varphi V_c$, theoretically, no shear reinforcement is required.
2. Where $\varphi V_c \geq V_u \geq 0.5\varphi V_c$, provide $A_{v,min}$ *satisfying Table* 9.6.3.4 *and Table* 9.7.6.2.2.
3. Where $V_u > \varphi V_c$, provide A_v *satisfying Sections* 22.5.8.1 *and* 22.5.8.5.3 *and Table* 9.7.6.2.2.

From centerline (ft)	V_u (kips)	V_c (kips)	ϕV_c (kips)	$0.5\phi V_c$ (kips)	Shear Reinforcement per ACI	Adopted by EOR
1	88.2	52.6	40	20	Av is required	A_v
2.46 (h/2 from face)	83.6	52.6	40	20	Av is required	A_v
3	81.9	52.6	40	20	Av is required	A_v
5	75.5	66.7	51	25.5	Av is required	A_v
7	69.2	131.5	99	49.5	Av,min is required	$A_{v,min}$
9	62.8	131.5	99	49.5	Av,min is required	$A_{v,min}$
11	56.5	131.5	99	49.5	Av,min is required	$A_{v,min}$
13	50.1	131.5	99	49.5	Av,min is required	$A_{v,min}$
15	43.8	122.9	93	46.5	None Required	$A_{v,min}$
17	37.5	88.6	67	33.5	Av,min is required	$A_{v,min}$
19	31.1	68	51	25.5	Av,min is required	$A_{v,min}$
21	24.8	53.5	41	20.5	Av,min is required	$A_{v,min}$
23	18.4	52.6	40	20	None Required	$A_{v,min}$
25	12.1	52.6	40	20	None Required	$A_{v,min}$
27	5.7	52.6	40	20	None Required	$A_{v,min}$
29	0.7	52.6	40	20	None Required	$A_{v,min}$
31	7.1	52.6	40	20	None Required	$A_{v,min}$
33	13.4	52.6	40	20	None Required	$A_{v,min}$
35	19.7	52.6	40	20	None Required	$A_{v,min}$
37	26.1	56.8	43	21.5	Av,min is required	$A_{v,min}$
39	32.4	72.6	55	27.5	Av,min is required	$A_{v,min}$
41	38.8	96.1	73	36.5	Av,min is required	$A_{v,min}$
43	45.1	131.5	99	49.5	None Required	$A_{v,min}$
45	51.5	131.5	99	49.5	Av,min is required	$A_{v,min}$
47	57.8	131.5	99	49.5	Av,min is required	$A_{v,min}$
49	64.2	131.5	99	49.5	Av,min is required	$A_{v,min}$
51	70.5	121.9	92	46	Av,min is required	$A_{v,min}$
53	76.8	64.7	49	24.5	Av is required	A_v
55	83.2	52.6	40	20	Av is required	A_v
57	89.5	52.6	40	20	Av is required	A_v
59	95.9	52.6	40	20	Av is required	A_v
59.54	97.6	52.6	40	20	Av is required	A_v
61	102.2	52.6	40	20	Av is required	A_v

Per ACI 318-19 Table 9.7.6.2.2, maximum spacing of legs of reinforcement for prestressed beam, measured along the beam's length:

	Maximum spacing, s (in)
$Required\ V_s \leq 4\sqrt{f'_c}b_w d$	**3h/4 = 26.25 inches**
	24 inches
$Required\ V_s > 4\sqrt{f'_c}b_w d$	**3h/8 = 13.125 inches**
	12

Per ACI 318-19 Section 22.8.5.1, at each section where $V_u > \varphi V_c$, transverse reinforcement shall be provided such that the following is satisfied:

$$V_s \geq \frac{V_u}{\varphi} - V_c$$

Per ACI 318-19 Section 22.5.8.5.3, shear reinforcement shall be calculated by:

$$V_s = \frac{A_v f_{yt} d}{s}$$

Therefore:

$$\frac{A_v}{s}(\text{reqd}) = \frac{V_u - \varphi V_c}{\varphi f_{yt} d}$$

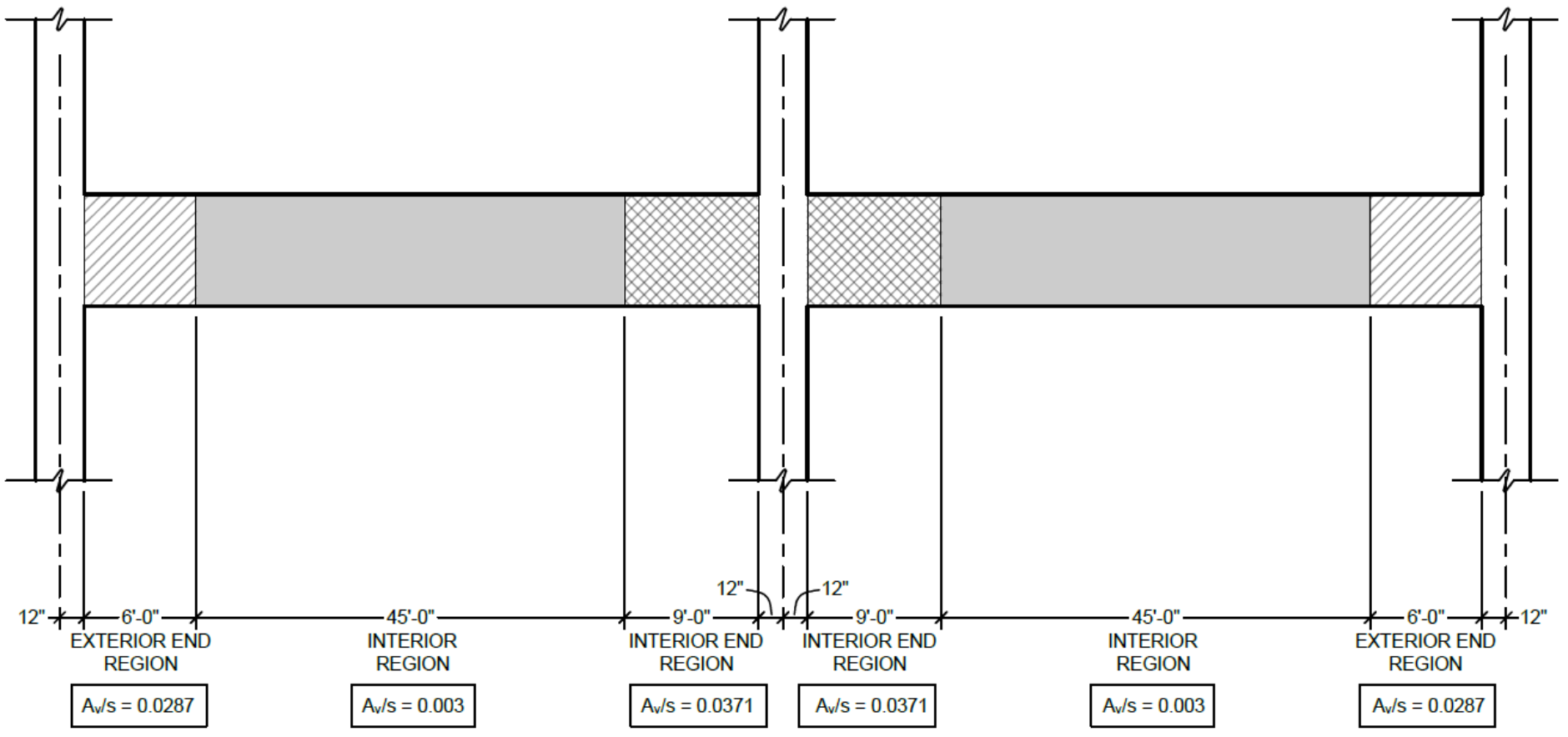
From centerline (ft)	V_u (kips)	V_c (kips)	ϕV_c (kips)	$0.5\phi V_c$ (kips)	A_v/s required	V_s (kips, minimum)	A_v/s selected by EOR
1	88.2	52.6	40	20	0.0287	65	0.0287
2.46 (h/2 from face)	83.6	52.6	40	20	0.026	58.9	0.0287
3	81.9	52.6	40	20	0.025	56.6	0.0287
5	75.5	66.7	51	25.5	0.0146	34	0.0287
7	69.2	131.5	99	49.5	0.003	Not Applicable	0.0287
9	62.8	131.5	99	49.5	0.003	Not Applicable	0.003
11	56.5	131.5	99	49.5	0.003	Not Applicable	0.003
13	50.1	131.5	99	49.5	0.003	Not Applicable	0.003
15	43.8	122.9	93	46.5	0.003	Not Applicable	0.003
17	37.5	88.6	67	33.5	0.003	Not Applicable	0.003
19	31.1	68	51	25.5	0.003	Not Applicable	0.003
21	24.8	53.5	41	20.5	0.003	Not Applicable	0.003
23	18.4	52.6	40	20	0.003	Not Applicable	0.003
25	12.1	52.6	40	20	0.003	Not Applicable	0.003
27	5.7	52.6	40	20	0.003	Not Applicable	0.003
29	0.7	52.6	40	20	0.003	Not Applicable	0.003
31	7.1	52.6	40	20	0.003	Not Applicable	0.003
33	13.4	52.6	40	20	0.003	Not Applicable	0.003
35	19.7	52.6	40	20	0.003	Not Applicable	0.003
37	26.1	56.8	43	21.5	0.003	Not Applicable	0.003
39	32.4	72.6	55	27.5	0.003	Not Applicable	0.003
41	38.8	96.1	73	36.5	0.003	Not Applicable	0.003
43	45.1	131.5	99	49.5	0.003	Not Applicable	0.003
45	51.5	131.5	99	49.5	0.003	Not Applicable	0.003
47	57.8	131.5	99	49.5	0.003	Not Applicable	0.003
49	64.2	131.5	99	49.5	0.003	Not Applicable	0.003
51	70.5	121.9	92	46	0.003	Not Applicable	0.0371
53	76.8	64.7	49	24.5	0.0166	37.7	0.0371
55	83.2	52.6	40	20	0.0258	58.4	0.0371
57	89.5	52.6	40	20	0.0295	66.8	0.0371
59	95.9	52.6	40	20	0.0333	75.3	0.0371
59.54	97.6	52.6	40	20	0.0343	77.6	0.0371
61	102.2	52.6	40	20	0.0371	83.7	0.0371

Note that in all cases:

$V_s < 4\sqrt{f'_c}b_w d = 105$ kips, so maximum spacing is 24" on center per Table 9.7.6.2.2 is 24" on center.

Step 6 – Shear reinforcement design results

The engineer has selected A_v/s (in^2/in) ratios that result in three different regions along the length of the beam. Theoretically, the EOR could present the shear reinforcement requirement on the CDs in terms of required A_v/s ratio regions as shown below (accompanied by a maximum prescriptive spacing as established above), and then let the WWR detailer handle the rest.



In Part 2 of this WRI blog entry, we will illustrate how the above "raw" shear reinforcement requirements will be configured in the form of a WWR solution that optimizes steel material to suit the specific design.

For more information on WWR, refer to www.wirereinforcementinstitute.org.