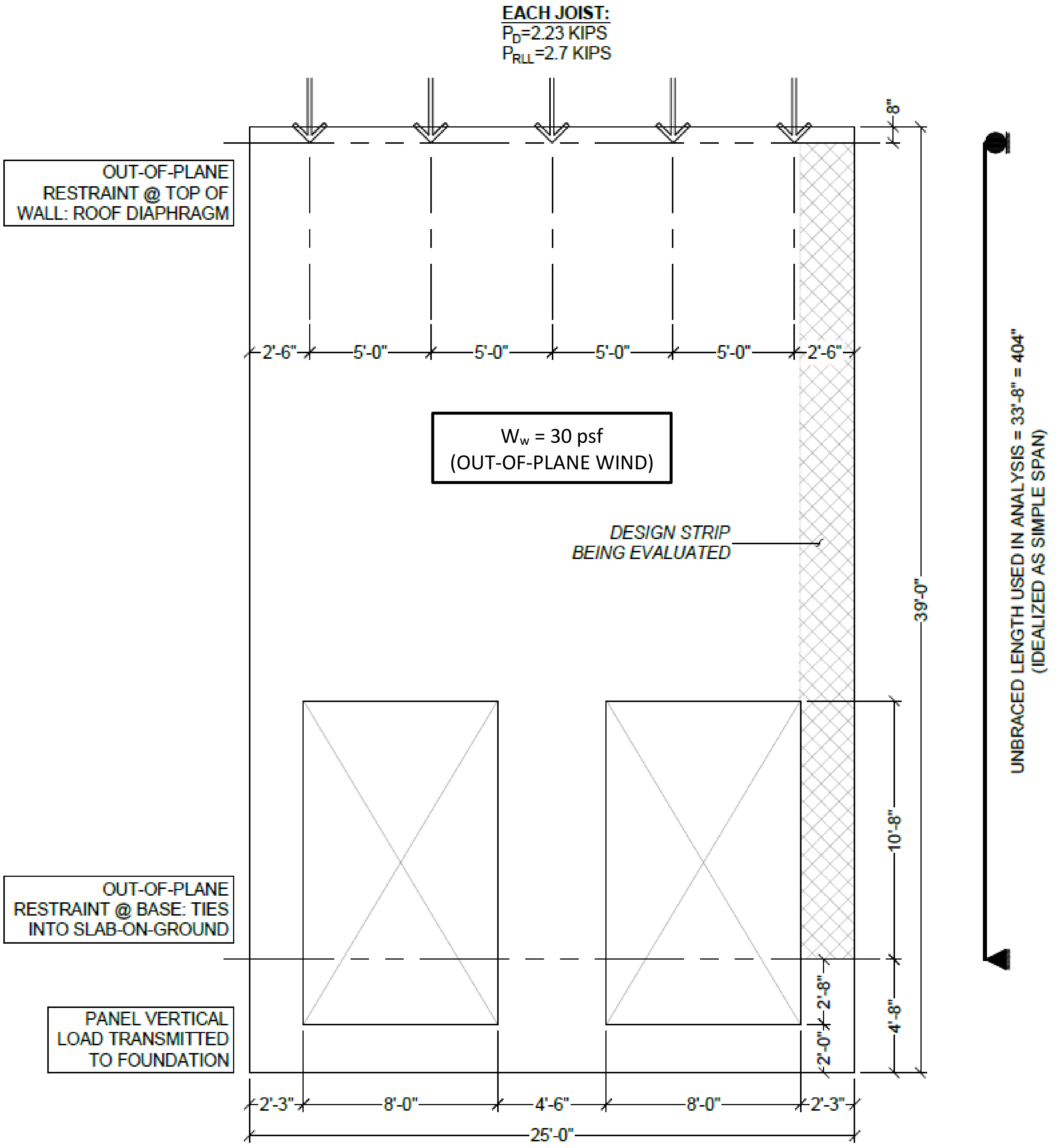
The ACI Alternative Method for Out-of-Plane Slender Wall Analysis: What is the Impact of a High Yield Strength / Reduced Reinforcement Area Substitution?



The image above shows a typical reinforced concrete tilt-up wall panel configured with two loading dock doors and supporting long-span roof framing. The wall is to be analyzed and designed per ACI 318-19 Section 11.8 “Alternative Method for Out-of-Plane Slender Wall Analysis”. In carrying out the analysis, we will explore the impact of a downstream substitution of “equivalent capacity” vertical reinforcement, characterized by higher yield strength but a proportionally smaller steel area.

I. DESIGN PARAMETERS

- Panel thickness = 9.5 inches
- Structural thickness, h = 8.75 inches (9.5” minus 0.75” architectural reveal feature)
- Two mats of reinforcement, one each face
- Clear cover to vertical reinforcing wires = 1.0”, each face
- 28-day compressive strength of concrete = 4 ksi ($\beta_1 = 0.85$)
- Steel reinforcement yield strength = 60 ksi
- Concrete unit weight = 150 pcf
- Unbraced length of design strip, l_c = 33.67 ft = 404 inches
- Width of design strip, l_w = 2.25 ft = 27 inches
- Joist end reactions are applied at a 3-inch horizontal eccentricity from the vertical centerline of wall thickness

II. LOAD DISTRIBUTION TO UNIT STRIP

Total joist dead load = (5)(2.23 kips) = 11.2 kips
Total distribution width = 25’-0”
Joist dead load distribution = 11.2 kips / 25’ = 0.448 kip/ft
Joist dead load to design strip = (0.448 kip/ft)(2.25’ + 8.0’/2) = 2.8 kips

Total joist live load = (5)(2.70 kips) = 13.5 kips
Total distribution width = 25’-0”
Joist live load distribution = 13.5 kips / 25’ = 0.540 kip/ft
Joist live load to design strip = (0.540 kip/ft)(2.25’ + 8.0’/2) = 3.4 kips

Total wall dead load, taken at “midheight” of design strip unbraced length:
(0.150 pcf)(9.5”/12)(33.67’/2 + 0.67’)(25’) = 52 kips
Wall dead load to design strip = 52 kips / 25’ x (2.25’ + 8.0’/2) = 13 kips

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↓ STRENGTH CHECK – STEPS III THROUGH VI ↓

III. APPLIED LOADING

All relevant code-prescribed load combinations should be checked in the design of tilt-up wall panels. For this example, it has been determined that the governing <u>strength level</u> load combination is 1.2D + 1.0W + 1.0L + 0.5Lr. This combination simplifies to 1.2D + 1.0W + 0.5Lr since there is no floor live load supported by the wall panel.

Load combination for strength check: 1.2D + 1.0W + 0.5Lr
Portion of axial load applied with a horizontal eccentricity:
P_{ua} = 1.2(2.8 kips) + 0.5(3.4 kips) = 5.1 kips

Total axial load at midheight:
P_{um} = 5.1 kips + 1.2(13 kips) = 20.7 kips

Out-of-plane wind on design strip and supported tributary width:
w_u = 1.0(30 psf)(2.25' + 8'/2) = 0.188 kip/ft

Moment at mid-height of design strip due to factored lateral and eccentric vertical loads, not including P-Δ effects:

$$M_{ua} = \frac{w_u l_c^{\,2}}{8} + \frac{P_{ua} e_{cc}}{2}$$

$$M_{ua} = \frac{(0.188)(33.67)^2}{8} + \frac{(5.1)(3"/12)}{2} = 27.3\ kip - ft$$

IV.TRIAL REINFORCEMENT AND INITIAL CHECKS

Assume a vertical steel reinforcement area A_s = 1.25 in^2.

Check #1: Minimum vertical reinforcement (ACI 318-19 Table 11.6.1)

$$\rho_l = \frac{A_s}{bt} = \frac{1.25}{27" \times 9.5"} = 0.0049 > 0.0010\ ACI\ minimum$$

Check #2: ACI Prescriptive spacing

Number of wires	Approximate center-to-center spacing*
5	5.25"
6	4.2"
7	3.5"

**Assumes 3" distance to wire centerline, each edge of design strip.*

Maximum spacing (ACI 318-19 Section 11.7.2.2) lesser of (5)(h=8.75") = 43.75 and 18". All options listed satisfy the requirement.

Minimum spacing (ACI 318-19 Section 25.2.3) shall be the greatest of 1.5", 1.5d_b, and (4/3)d_{agg}. By inspection, even if 7 wires are used, minimum spacing requirement is easily satisfied.

Check #3: Maximum axial stress at midheight, ACI 318-19 Section 11.8.1.1(d)

$$\frac{P_{um}}{A_g} = \frac{20.7\ kips}{8.75" \times 27"} = 0.088\ ksi < 0.06f'_c = 0.24\ ksi\ \therefore OK!$$

Check #4: confirm tension-controlled behavior for out-of-plane moment effect, ACI 318-19 Section 11.8.1.1(b)

Assume maximum wire size = 5/8" diameter, therefore d = 8.75" – 1" – 0.625"/2 = 7.44"

Effective area of longitudinal reinforcement, $A_{se,w}$ (ACI 318-19 Section R11.8.3.1):

$$A_{se,w} = A_s + \frac{P_{um}}{f_y}\left(\frac{h/2}{d}\right) = 1.25 + \frac{20.7\ kips}{60\ ksi}\left(\frac{8.75"/2}{7.44"}\right) = 1.45\ in^2$$

$$a = \frac{A_{se} f_y}{0.85 f'_c b} = \frac{1.45\ in^2 \times 60\ ksi}{.85 \times 4\ ksi \times 27"} = 0.948"$$

$$c = a/\beta_1 = 1.115"$$

$$\varepsilon_s = \frac{0.003 \beta_1 d}{a} - 0.003 = \frac{.003 \times 0.85 \times 7.44"}{0.948} - 0.003 = 0.017\ in/in \gg \varepsilon_{ty} + 0.003 = 0.0051\ in/in$$

$\therefore$ *Section is tension controlled.* $\varphi = 0.90$.

V. CHECK DESIGN MOMENT STRENGTH (ϕMn) ≥ CRACKING MOMENT (M_{cr}) AS REQUIRED BY ACI 318.19 SECTION 11.8.1.1(C)

$$\varphi M_n = \varphi A_{se} f_y \left(d - \frac{a}{2}\right) = 0.9 \times 1.45\ in^2 \times 60\ ksi \times \left(7.44" - \frac{0.948"}{2}\right) = 45.5\ kip \cdot ft$$

$$M_{cr} = \frac{f_r I_g}{h/2} = \frac{7.5\sqrt{4{,}000} \times \frac{27" \times 8.75"^3}{12}}{8.75"/2} = 163.43\ kip \cdot in = 13.62\ kip \cdot ft$$

$\therefore\ \varphi M_n > M_{cr}$

VI. CHECK DESIGN MOMENT STRENGTH (ϕMn) ≥ MAXIMUM FACTORED MOMENT (Mu) AS REQUIRED BY ACI 318.19 SECTION 11.5.1.1(b)

$\varphi M_n = 45.5\ kip \cdot ft$ *per previous calculation*

M_u by direct calculation using ACI 318-19 Section 11.8.3.1(b):

$$M_u = \frac{M_{ua}}{\left(1 - \frac{5 P_u {l_c}^2}{0.75 \times 48 E_c I_{cr}}\right)}$$

M_{ua} = 27.3 kip-ft from Step III.

P_u = P_{um} = 20.7 kips from Step III.

l_c = 33.67 ft = 404 inches from Step I.

$E_c = 57{,}000 \times \sqrt{f'c} = 3{,}605\ ksi$ per ACI 318-19 Section 19.2.2.1(b)

$$I_{cr} = \frac{E_s}{E_c}\left(A_{se,w}\right)(d-c)^2 + \frac{l_w c^3}{3} = \frac{29{,}000}{3{,}605} \times 1.45 \times (7.44 - 1.115)^2 + \frac{27 \times 1.115^3}{3}$$

$I_{cr} = 479.1\ in^4$

$\therefore\ M_u = 37.5\ kip \cdot ft$

STRENGTH CHECK RESULTS
$\varphi M_n = 45.5\ kip \cdot ft\ > M_u = 37.5\ kip \cdot ft$

↓ **SERVICE DEFLECTION CHECK**
STEPS VII THROUGH VIII ↓

VII. APPLIED LOADING

Per discussion in ACI 318-19 Section R11.8.4.1, we will use the service-level load combination D + 0.5L + W_a for calculating service level lateral deflection. The equation manifests as follows since (a) there is no floor live load supported by the wall panel and (b) we utilize the 10-year mean return interval wind speed for the purposes of determining deflection limits defined in the International Building Code (IBC):

Load combination for service deflection check: 1.0D + 0.42W_a

Portion of axial load applied with a horizontal eccentricity:
P_a = 1.0(2.8 kips) = 2.8 kips

Total axial load at midheight:
P_s = 2.8 kips + 1.0(13 kips) = 15.8 kips

Out-of-plane wind on design strip and supported tributary width:
w_u = 0.42(30 psf)(2.25' + 8'/2) = 0.079 kip/ft

Moment at mid-height of design strip due to service-level lateral and eccentric vertical loads, not including P-Δ effects:

$$M_{sa} = \frac{w_s l_c{}^2}{8} + \frac{P_a e_{cc}}{2}$$

$$M_{sa} = \frac{(0.079)(33.67)^2}{8} + \frac{(2.8)(3"/12)}{2} = 11.54\, kip - ft$$

Keep in mind that per ACI 318-19 Section 11.8.4.2, the maximum moment M_a at midheight of wall due to service lateral and eccentric vertical loads, including $P_s\Delta_s$ effects, is:

$$M_a = M_{sa} + P_s\Delta_s$$

VIII. ITERATION OF DEFLECTION

We now refer to ACI 318-19 Table 11.8.4.1 for the applicable calculation of the out-of-plane deflection due to service loads, Δ_s.

→ Is $M_a \le (2/3)M_{cr}$?

To answer this question, as a starting trial value we will assume that the previously calculated value of M_{sa} is representative of the M_a value. With that in mind:

$$M_{a,trial} = 11.54\, kip \cdot ft > 2/3 \times 13.62\, kip \cdot ft = 9.08\, kip \cdot ft$$

$\therefore$ *initially assume the relationship* $\Delta_s = (2/3)\Delta_{cr} + \dfrac{(M_a - (2/3)M_{cr})}{(M_n - (2/3)M_{cr})} \times \left(\Delta_n - (2/3\Delta_{cr})\right)$

$$\Delta_{cr} = \frac{5M_{cr}{l_c}^2}{48E_cI_g} = \frac{5 \times 13.62\ kip \cdot ft \times 12 \times 404"^2}{48 \times 3{,}605 \times 1/12 \times 27" \times 8.75"^3} = 0.511"$$

M_{sa} and Δ_{cr} are constants. But we also need M_n and I_{cr} - both of which are required for the Δ_n calculation - and both of which are strength level attributes. As such, for each service load combination we are checking (only one in this example), the "corresponding" strength load combination must be used to get a portion of the information necessary for calculation of service load deflection.

The strength load combination that corresponds to the service load combination we are checking in this example is as follows:

Service Load Combination Basic Form	Corresponding Strength Load Combination Basic Form
1.0D + 0.5L + W_a (R11.8.4.1)	1.2D + 1.0L + 1.0W (5.3.1d)

Calculate M_n and I_{cr}.

$$A_{se} = A_s + \frac{P_u}{f_y}\left(\frac{h/2}{d}\right) = 1.25 + \left(\frac{1.2 \times (2.8\ kips + 13\ kips)}{60\ ksi}\right)\left(\frac{8.75"/2}{7.44"}\right) = 1.436\ in^2$$

$$a = \frac{A_{se}f_y}{0.85f'_cb} = \frac{1.436\ in^2 \times 60\ ksi}{.85 \times 4\ ksi \times 27"} = 0.939"$$

$$c = a/\beta_1 = 1.105"$$

$$M_n = A_{se}f_y\left(d - \frac{a}{2}\right) = 1.436\ in^2 \times 60\ ksi \times \left(7.44" - \frac{0.939"}{2}\right) = 50\ kip \cdot ft$$

$$I_{cr} = \frac{E_s}{E_c}(A_{se})(d-c)^2 + \frac{l_wc^3}{3} = \frac{29{,}000}{3{,}605} \times 1.436 \times (7.44 - 1.105)^2 + \frac{27 \times 1.105^3}{3}$$

$$I_{cr} = 475.5\ in^4$$

$$\Delta_n = \frac{5M_n{l_c}^2}{48E_cI_{cr}} = \frac{5 \times 50 \times 12 \times 404^2}{48 \times 3605 \times 475.5} = 5.95"$$

Iteration #1

$$\Delta_s = (2/3)\Delta_{cr} + \frac{(M_a - (2/3)M_{cr})}{(M_n - (2/3)M_{cr})} \times \left(\Delta_n - (2/3\Delta_{cr})\right)$$

$$\Delta_s = (2/3)(0.511) + \frac{(\mathbf{11.54} - (2/3)13.62)}{(50 - (2/3)13.62)} \times \left(5.95 - (2/3 \times 0.511)\right) = 0.678"$$

$$M_a = M_{sa} + P_s\Delta_s \rightarrow 11.54\ kip \cdot ft + 15.8\ kips \times \frac{0.678"}{12} = \mathbf{12.43}\ kip \cdot ft$$

$\mathbf{11.54} \neq \mathbf{12.43}$ $\therefore$ *solution has not converged.*

$\mathbf{12.43}\ kip \cdot ft \downarrow$

Iteration #2

$$\Delta_s = (2/3)(0.511) + \frac{(12.43 - (2/3)\,13.62)}{(50 - (2/3)13.62)} \times \left(5.95 - (2/3 \times 0.511)\right) = 0.800"$$

$$M_a = M_{sa} + P_s\Delta_s \rightarrow 11.54\ kip \cdot ft + 15.8\ kips \times \frac{0.800"}{12} = 12.59\ kip \cdot ft$$

$12.43 \neq 12.59\ \therefore$ *solution has not converged.*

$12.59\ kip \cdot ft \downarrow$

Iteration #3

$$\Delta_s = (2/3)(0.511) + \frac{(12.59 - (2/3)13.62)}{(50 - (2/3)13.62)} \times \left(5.95 - (2/3 \times 0.511)\right) = 0.822"$$

$$M_a = M_{sa} + P_s\Delta_s \rightarrow 11.54\ kip \cdot ft + 15.8\ kips \times \frac{0.822"}{12} = 12.62\ kip \cdot ft$$

$12.59 \approx 12.62\ \therefore$ *solution has converged.*

SERVICE LEVEL RESULTS
$M_a = 12.62\ kip \cdot ft > 9.08\ kip \cdot ft\ \therefore$ *initial assumption OK*
$\Delta_s = 0.822" < l_c/150 = 2.69"$ per ACI 318-19 Section 11.8.1.1(e) $\therefore OK$

IX. SELECT VERTICAL REINFORCEMENT

We have confirmed that both strength and deflection are satisfactory. The following reinforcement options, then, are acceptable:

REINFORCEMENT OPTIONS (ANALYZED AT 60 KSI YIELD STRENGTH)	
Number of wires	**Wire Size to provide A_s = 1.25 in^2**
5	D25.0
6	D20.9
7	D17.9

Now let’s assume that a “value-add” alternative is proposed wherein the originally specified 60 ksi reinforcement is to be replaced with 80 ksi wires, but with the cross-sectional area of steel reduced proportionally.

$$\left(A_s f_y\right)_{original} = \left(A_s f_y\right)_{replacement}$$

$$(1.25 \times 60)_{original} = (A_s \times 80)_{replacement}$$

$$A_{s,replacement} = 0.938\ in^2 \rightarrow so\ assume\ (5)\ D18.8\ in\ lieu\ of\ (5)\ D25.0.$$

How does this impact the original design?

↓ STRENGTH CHECK – STEPS III THROUGH VI ↓

III. APPLIED LOADING

<UNCHANGED>

P_{ua} = 1.2(2.8 kips) + 0.5(3.4 kips) = 5.1 kips

P_{um} = 5.1 kips + 1.2(13 kips) = 20.7 kips

w_u = 1.0(20 psf)(2.25' + 8'/2) = 0.125 kip/ft

$$M_{ua} = \frac{(0.125)(33.67)^2}{8} + \frac{(5.1)(3"/12)}{2} = 18.36\, kip - ft$$

IV. TRIAL REINFORCEMENT AND INITIAL CHECKS

Assume a vertical steel reinforcement area A_s = 0.938 in^2.

Check #1: Minimum vertical reinforcement (ACI 318-19 Table 11.6.1)

$$\rho_l = \frac{A_s}{bt} = \frac{0.938}{27" \times 9.5"} = 0.0037 > 0.0010\ ACI\ minimum$$

Check #2: ACI Prescriptive spacing

<UNCHANGED>

$OK!$

Check #3: Maximum axial stress at midheight, ACI 318-19 Section 11.8.1.1(d)

<UNCHANGED>

$OK!$

Check #4: confirm tension-controlled behavior for out-of-plane moment effect, ACI 318-19 Section 11.8.1.1(b)

Maintain use of d = 7.44".

Effective area of longitudinal reinforcement, $A_{se,w}$ (ACI 318-19 Section R11.8.3.1):

$$A_{se,w} = A_s + \frac{P_{um}}{f_y}\left(\frac{h/2}{d}\right) = .938 + \frac{20.7\ kips}{80\ ksi}\left(\frac{8.75"/2}{7.44"}\right) = 1.09\ in^2$$

$$a = \frac{A_{se}f_y}{0.85f'_c b} = \frac{1.09\ in^2 \times 80\ ksi}{.85 \times 4\ ksi \times 27"} = 0.949"$$

$$c = a/\beta_1 = 1.116"$$

$$\varepsilon_s = \frac{0.003\beta_1 d}{a} - 0.003 = \frac{.003 \times 0.85 \times 7.44"}{0.949} - 0.003 = 0.017\ in/in \gg \varepsilon_{ty} + 0.003 = 0.0051\ in/in$$

$\therefore$ *Section is tension controlled.* $\varphi = 0.90$.

V. CHECK DESIGN MOMENT STRENGTH (ϕMn) ≥ CRACKING MOMENT (Mcr) AS REQUIRED BY ACI 318.19 SECTION 11.8.1.1(C)

$$\varphi M_n = \varphi A_{se} f_y \left(d - \frac{a}{2}\right) = 0.9 \times 1.09\ in^2 \times 80\ ksi \times \left(7.44" - \frac{0.949"}{2}\right) = 45.6\ kip \cdot ft$$

Note that the design moment strength is unchanged (slight rounding disparity not withstanding). This only makes sense as the tensile force product $A_s f_y$ is unchanged.

$M_{cr} = 13.62\ kip \cdot ft$ **<UNCHANGED>**

$\therefore\ \varphi M_n > M_{cr}$

VI. CHECK DESIGN MOMENT STRENGTH (ϕMn) ≥ MAXIMUM FACTORED MOMENT (Mu) AS REQUIRED BY ACI 318.19 SECTION 11.5.1.1(b)

$\varphi M_n = 45.6\ kip \cdot ft\ per\ previous\ calculation$

M_u by direct calculation using ACI 318-19 Section 11.8.3.1(b):

$$M_u = \frac{M_{ua}}{\left(1 - \frac{5 P_u {l_c}^2}{0.75 \times 48 E_c I_{cr}}\right)}$$

M_{ua} = 27.3 kip-ft from Step III. **<UNCHANGED>**

$P_u = P_{um}$ = 20.7 kips from Step III. **<UNCHANGED>**

l_c = 33.67 ft = 404 inches from Step I. **<UNCHANGED>**

$E_c = 57{,}000 \times \sqrt{f'c} = 3{,}605\ ksi$ per ACI 318-19 Section 19.2.2.1(b) **<UNCHANGED>**

$$I_{cr} = \frac{E_s}{E_c}\left(A_{se,w}\right)(d - c)^2 + \frac{l_w c^3}{3} = \frac{29{,}000}{3{,}605} \times 1.09 \times (7.44 - 1.116)^2 + \frac{27 \times 1.116^3}{3}$$

$I_{cr} = 363.2\ in^4 \leftarrow$ ***this is a significant reduction compared to original calculation.***

$\therefore\ M_u = 42.5\ kip \cdot ft$

Notice the increase in the flexural demand. This is because, despite utilizing an “equal strength” of reinforcement, the reduced cross-sectional area of reinforcing steel reduces the cracked section moment of inertia. A reduced moment of inertia corresponds to an increase in the out-of-plane deflection of the wall, which in turn results in an increase in the P-Δ effect that serves to magnify the flexural demand. The demand-to-capacity ratio of the design strip is still satisfactory in this example, but the increase in demand (roughly 13%) is not negligible and must always be checked in a scenario where an “equal strength” alternative consisting of a lesser steel area is being considered for use.

STRENGTH CHECK RESULTS

$\varphi M_n = 45.6\ kip \cdot ft\ > M_u = 42.5\ kip \cdot ft$

↓ SERVICE DEFLECTION CHECK ↓
STEPS VII THROUGH VIII

VII. APPLIED LOADING

<UNCHANGED>

Portion of axial load applied with a horizontal eccentricity:
P_a = 1.0(2.8 kips) = 2.8 kips

Total axial load at midheight:
P_s = 2.8 kips + 1.0(13 kips) = 15.8 kips

Out-of-plane wind on design strip and supported tributary width:
w_u = 0.42(30 psf)(2.25' + 8'/2) = 0.079 kip/ft

Moment at mid-height of design strip due to service-level lateral and eccentric vertical loads, <u>not including P-Δ effects</u>:

$$M_{sa} = \frac{(0.079)(33.67)^2}{8} + \frac{(2.8)(3"/12)}{2} = 11.54\, kip - ft$$

$$M_a = M_{sa} + P_s\Delta_s$$

VIII. ITERATION OF DEFLECTION

We now refer to ACI 318-19 Table 11.8.4.1 for the applicable calculation of the out-of-plane deflection due to service loads, Δ_s.

→ Is $M_a \leq (2/3)M_{cr}$?

To answer this question, as a starting trial value we will assume that the previously calculated value of M_{sa} is representative of the M_a value. With that in mind:

$$M_{a,trial} = 11.54\, kip \cdot ft > 2/3 \times 13.62\, kip \cdot ft = 9.08\, kip \cdot ft$$ **<UNCHANGED>**

$$\therefore\ initially\ assume\ the\ relationship\ \Delta_s = (2/3)\Delta_{cr} + \frac{(M_a - (2/3)M_{cr})}{(M_n - (2/3)M_{cr})} \times \left(\Delta_n - (2/3\Delta_{cr})\right)$$

$\Delta_{cr} = 0.511"$ **<UNCHANGED>**

Calculate M_n and I_{cr}.

$$A_{se} = A_s + \frac{P_u}{f_y}\left(\frac{h/2}{d}\right) = 0.938 + \left(\frac{1.2 \times (2.8\, kips + 13\, kips)}{80\, ksi}\right)\left(\frac{8.75"/2}{7.44"}\right) = 1.077\, in^2$$

$$a = \frac{A_{se}f_y}{0.85f'_c b} = \frac{1.077\, in^2 \times 80\, ksi}{.85 \times 4\, ksi \times 27"} = 0.939"$$

$c = a/\beta_1 = 1.105"$

$$M_n = A_{se} f_y \left(d - \frac{a}{2}\right) = 1.077\ in^2 \times 80\ ksi \times \left(7.44" - \frac{0.939"}{2}\right) = 50\ kip \cdot ft$$

Note that M_n is unchanged.

$$I_{cr} = \frac{E_s}{E_c} (A_{se})(d-c)^2 + \frac{l_w c^3}{3} = \frac{29{,}000}{3{,}605} \times 1.077 \times (7.44 - 1.105)^2 + \frac{27 \times 1.105^3}{3}$$

$I_{cr} = 359.8\ in^4 \leftarrow \boldsymbol{this\ is\ a\ significant\ reduction\ compared\ to\ original\ calculation}.$

$$\Delta_n = \frac{5 M_n {l_c}^2}{48 E_c I_{cr}} = \frac{5 \times 50 \times 12 \times 404^2}{48 \times 3605 \times 359.8} = 7.86"$$

Iteration #1

$$\Delta_s = (2/3)\Delta_{cr} + \frac{(M_a - (2/3)M_{cr})}{(M_n - (2/3)M_{cr})} \times \left(\Delta_n - (2/3\Delta_{cr})\right)$$

$$\Delta_s = (2/3)(0.511) + \frac{(\mathbf{11.54} - (2/3)13.62)}{(50 - (2/3)13.62)} \times \left(7.86 - (2/3 \times 0.511)\right) = 0.793"$$

$$M_a = M_{sa} + P_s\Delta_s \rightarrow 11.54\ kip \cdot ft + 15.8\ kips \times \frac{0.793"}{12} = \mathbf{12.58}\ kip \cdot ft$$

$\mathbf{11.54} \neq \mathbf{12.58}\ \therefore solution\ has\ not\ converged.$

$\mathbf{12.58}\ kip \cdot ft \downarrow$

Iteration #2

$$\Delta_s = (2/3)(0.511) + \frac{(\mathbf{12.58} - (2/3)13.62)}{(50 - (2/3)13.62)} \times \left(7.86 - (2/3 \times 0.511)\right) = 0.984"$$

$$M_a = M_{sa} + P_s\Delta_s \rightarrow 11.54\ kip \cdot ft + 15.8\ kips \times \frac{0.984"}{12} = \mathbf{12.84}\ kip \cdot ft$$

$\mathbf{12.58} \neq \mathbf{12.84}\ \therefore solution\ has\ not\ converged.$

$\mathbf{12.84}\ kip \cdot ft \downarrow$

Iteration #3

$$\Delta_s = (2/3)(0.511) + \frac{(\mathbf{12.84} - (2/3)13.62)}{(50 - (2/3)13.62)} \times \left(7.86 - (2/3 \times 0.511)\right) = 1.032"$$

$$M_a = M_{sa} + P_s\Delta_s \rightarrow 11.54\ kip \cdot ft + 15.8\ kips \times \frac{1.032"}{12} = \mathbf{12.90}\ kip \cdot ft$$

$\mathbf{12.84} \neq \mathbf{12.90}\ \therefore solution\ has\ not\ converged.$

$\mathbf{12.90}\ kip \cdot ft \downarrow$

Iteration #4

$$\Delta_s = (2/3)(0.511) + \frac{(\mathbf{12.90} - (2/3)13.62)}{(50 - (2/3)13.62)} \times \left(7.86 - (2/3 \times 0.511)\right) = 1.043"$$

$$M_a = M_{sa} + P_s\Delta_s \rightarrow 11.54\ kip \cdot ft + 15.8\ kips \times \frac{1.043"}{12} = \mathbf{12.91}\ kip \cdot ft$$

$$\mathbf{12.90} \approx \mathbf{12.91}\ \therefore solution\ has\ converged.$$

SERVICE LEVEL RESULTS
$M_a = 12.91\ kip \cdot ft > 9.08\ kip \cdot ft\ \therefore initial\ assumption\ OK$ $\Delta_s = 1.043" < l_c/150 = 2.69"$ per ACI 318-19 Section 11.8.1.1(e) $\therefore OK$

While the resulting service-level deflection still falls within the acceptable range per ACI, it is noteworthy that it has increased approximately 27% simply as a consequence of utilizing a lesser area of higher yield strength reinforcement.

It is important to point out that this behavior is not unique to welded wire reinforcement. It is applicable to all mild steel reinforcement – including rebar – as it is based on a characteristic stiffness derived from the composite interaction of hardened concrete and the cross-sectional area of tensile steel embedded therein.

Due to the prevalence of 80 ksi WWR in the marketplace and its natural fit for use as reinforcement in tilt-up wall panels, designs should be based on the use of 80 ksi material from the start of the project, with cross-sectional steel areas established to satisfy strength and serviceability requirements. This effectively precludes any potential deflection and/or demand compatibility issues that might arise downstream in the event that a substitution is considered.

EXAMPLE SUMMARY		
ATTRIBUTE	**ORIGINAL DESIGN**	**ALTERNATIVE DESIGN**
Steel Yield Strength	60 ksi	80 ksi
Steel Area in Analysis	1.25 in^2	0.938 in^2
$A_s f_y$	75 kips	75 kips
Reinforcement Selected	(5) D25.0	(5) D18.8
ϕM_n	45.5 kip-ft	45.6 kip-ft
M_u (w/ P-Δ effect)	37.5 kip-ft	42.5 kip-ft (13% increase)
Strength Utilization Ratio	**0.824 < 1.0 OK**	**0.932 < 1.0 OK**
I_{cr}	475.5 in^4	359.8 in^4 (24% decrease)
Δ_s	0.822”	1.043” (27% increase)
% of “Allowable” Deflection	**31%**	**39%**

For more information on WWR, refer to www.wirereinforcementinstitute.org.